Introduction to Measurement and Error Analysis

(PHY 115 and 117)

Introduction
In the sciences, measurement plays an important role. The accuracy of the measurement, as well as the units, help scientists to better understand phenomena occurring in nature. The better time is measured, for example, the better we can calculate the vertical of a basketball player or the speed of an F1 racecar.

In this lab, we will use different instruments to measure the volume and mass of metal cylinders and calculate the error for each measurement, paying attention to significant figures as well. From these measurements, we will determine the density of each metal and compare them to the known values.

Equipment List
At least 3 cylinders of the same metal, 1 ruler, 1 set of vernier calipers, 1 micrometer, 1 graduated cylinder

Procedure
1. There will be at least three cylinders of the same metal at your lab station. Both you and your lab partner will independently use the ruler to measure the diameter (d) and length (L) of each cylinder. Record the minimum uncertainty for each measurement.

2. Repeat these measurements with a vernier caliper and, if possible, a micrometer. Record the minimum uncertainty. Again make sure that the measurements are independent of each other. The TA will show you how to use these instruments.

3. Make an average of the data for each instrument used.

4. Calculate the uncertainty of each of these measurements by taking the difference between you and your partner’s measurements and dividing them by two. Take the greater value between this calculation and the minimum uncertainty you recorded for your error of this measurement.

5. Measure the volume directly by immersing each metal cylinder into the graduated cylinder. Record both the initial and final volumes on the graduated cylinder as well as the uncertainty.

6. Determine the volume of each cylinder using the fact that

\[ V = \frac{\pi}{4} d^2 L \]  

and the averages from each instrument. Calculate the error for each volume as well.

7. Measure the mass m of each cylinder using a pan balance and record the uncertainty.
8. Calculate the density $\rho$ of each cylinder for each instrument used to measure the volume. Calculate the error for the density using the information in the Error and Uncertainty section of the lab manual. Density is mass divided by volume.

9. For each cylinder, take an average of the volumes that you calculated. Obtain the error for this average by taking the difference of the largest and smallest volume (without error included) and divide by two.

10. Plot mass vs. volume for each of the cylinders with error bars. Determine the best fit slope as given by the following Error and Uncertainty section with error. Compare this value with the values you calculated for density as well as the density given by the TA.

**Questions**
1. Are the values you calculated for the density within experimental uncertainty of the given value?

2. Were your values for density precise? Accurate? Both? Why or why not?

3. What assumptions have been made about the cylinders?
In physics, as well as other sciences, every measurement comes with some sort of uncertainty. In reporting the results of an experiment, it is necessary to report this uncertainty as well as the measured experimental value. This is just as true when measuring how fast Michael Johnson runs the 200 meters as measuring the radiation background in the universe. For this course, we are going to use a simple approach to estimate these uncertainties and calculate the overall error of an experiment.

One common misconception is that the experimental error is the difference between our measurement and the accepted (or “official”) value. What is meant by error is the estimate of the range of values within which the true value is likely to lie. The range is determined by the instruments used in the experiment and the methods we use.

**Error**

If we denote a quantity that is measured through an experiment as $X$, then the error is called $\Delta X$ (read as: delta X). If $X$ represents the time swam in the 100 meter freestyle, we might say the time is $t = 25.1 \pm 0.1$ s, where the central value for time is 25.1 s and the error $\Delta t = 0.1$ s. Both the central value and the error must be reported in your lab report. Note that in this example, the central value is given to three significant figures. Do not give figures beyond the first digit in your uncertainty for any quantity. They do not have any meaning and are therefore misleading.

In the example given in the previous paragraph, the error quoted for the time is called the **absolute error**. There is also the **relative error**, which is defined as the ratio of the error to the central value. The relative error for the 100 meter time is $\Delta t / t = 0.1 / 25.1 = 0.004$. Notice that the relative error has no dimensions and should have as many significant figures as those that are known for the absolute error.

**Random Error**

Random error occurs because of small variations in the measurement process. Measuring the time of a pendulum’s period with a stopwatch will give different times because of small differences in your reaction time of starting and stopping the stopwatch at the beginning and end of a period. If this is random error, then the average of the measurements will get closer to the correct value as the number of trials increases. The correct result would be the average for our central value and we will take the error to be the standard deviation of the measurements, in the following approximation. In this lab, we will rarely take many measurements of a quantity. Instead, a few measurements are taken (3-5). To get the central value, we would take the average of these measurements. To get the error, the difference is taken between the largest and smallest values and then divided by two. Remember to go only to the first significant figure for the error and only to that decimal place for the central value.

Example: A football player is training for the combine, a display of talents that will help teams determine if they want to draft him. In a week of training, he has five times for the 40: 4.3s, 4.2s, 4.4s, 4.5s, 4.2s. The average over the week is 4.32s. The error $\Delta t$ is calculated by $(4.5s - 4.2s) / 2 = .15s$. But because of the fact that we allow
error to have one significant figure, $\Delta t = 0.2s$. Thus, the average time with error is $t = 4.3s \pm 0.2s$.

**Systematic Error**
Some sources of error are not random. For example, if you used a meter stick to measure the length of a friend’s arm, the meter stick may be warped, or may have stretched, and you would never get a good value with that instrument. More subtly, the length of the meter stick might depend on temperature, and this measurement will be good only for the temperature that it was calibrated. In this course, systematic errors should be considered, but the effects in each lab are assumed to be small. However, if the value of a quantity seems to be far off from what you would expect, you should think about the possible sources of systematic error more carefully.

**Propagation of errors**
Often in the lab it is needed to combine two or more measured quantities, each of which has an error, to get a desired quantity. For example, you want to know the perimeter of a football field and you measured the length $l$ and the width $w$ with a tape measure. Since you know that the perimeter $p = 2(l + w)$, to get the error on $p$, you would need to know the errors estimated for $l$ and $w$, $\Delta l$ and $\Delta w$.

There are some rules for calculating errors of such combined, or derived, quantities. Suppose you have made primary measurements of $A$ and $B$, and would like to get the best value and error for some derived quantity $S$. For addition and subtraction of the given quantities:

- If $S = A + B$, then $\Delta S = \Delta A + \Delta B$.
- If $S = A - B$, then $\Delta S = \Delta A + \Delta B$ as well.

A more refined rule for either case $S = A \pm B$ is

$$\Delta S = \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

which takes into account the fact that some random error will cancel out.

For multiplication or division of measured quantities:

- If $S = A \times B$, then $\Delta S / S = \Delta A / A + \Delta B / B$.
- If $S = A / B$, then $\Delta S / S = \Delta A / A + \Delta B / B$ as well.

Note that these two take into account the relative errors of $A$ and $B$. Again, if the errors in $A$ and $B$ partly cancel out, you can use

$$\frac{\Delta S}{S} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Now an example needs to be done. For the equation for the perimeter above, $p = 2(l + w)$, the error is $\Delta p = 2\Delta l + 2\Delta w$. This is because 2 is a constant, so what you have is the case where $S = A + B$. Suppose you were interested also on the area of such a rectangle. Then, the error for the area ($Area = l \times w$) would be:

$$\Delta Area = Area \left(\frac{\Delta l}{l} + \frac{\Delta w}{w}\right)$$
**Obtaining Values from a Graph**

In some of these labs, you will be asked to graph data to obtain results from the slope of the graph. For an example of this, we will use the example of a drag racer. The data that we would normally take would be the distance \( l \) that the drag racer has gone in a time \( t \). From this data, \( l \) would be plotted on the \( y \) axis versus \( t^2 \) on the \( x \) axis (see figure below). The slope of this line would allow us to determine the acceleration that the drag racer experiences, \( l = \left(\frac{a \cdot t^2}{2}\right) \). In constructing the graph, plot a point at the central \((x, y)\) values for each of your measurements. Small horizontal bars should be drawn with a length that is equal to the absolute error in the \( x \) direction \((t^2)\) in both, the positive and negative directions. Similarly, small vertical bars should be drawn with a length that is equal to the absolute error in the \( y \) direction \((l)\) in both, the positive and negative directions.

Because of the error bars, we wish to find the slope that is most likely to fit the data. To do this, we draw two slopes. The first slope is drawn to pass through or near the upper error bars. The second is drawn to pass through or near the lower error bars. The average of these two slopes gives the best fit acceleration. Half the difference of these two slopes gives the absolute error of the acceleration.
Hooke’s Law

**Introduction**
Hooke’s Law relates the force exerted on a spring in relation to a small displacement from rest by a very simple relationship:

\[ F = -k \cdot x \]

where \( F \) is the force, \( x \) is the displacement from equilibrium, and \( k \) is the spring constant. We will be using masses hanging from a spring to determine this constant for the springs given in lab.

**Equipment**
One ring stand, one spring, various masses, one meter stick, one ruler

**Procedure**
Make sure that the meter stick is perpendicular to the table, so as not to have a systematic error.

Record the equilibrium position of the spring that is hanging on the ring stand. Use the ruler as a way to make sure that you’re recording the correct equilibrium point.

Hang one of the masses off the spring. Record the mass and the new equilibrium position of the spring once it has stopped oscillating.

Repeat with heavier masses.

Graph the weight of each mass versus \( x \), the displacement of the spring from its equilibrium position. The slope of this line will be the spring constant.

Repeat the procedure with one other set up.

Hang one large mass and record its distance and include it on your graph, but do not use it to calculate your slope.

**Questions**
1. Is there a linear relationship between the weight and displacement?

2. Are the values of the spring constant similar for the two springs that you measured? (Do you expect your results to be similar?) Why?

3. Does the heavy mass obey Hooke’s Law? Why or why not?
Standing Waves

Introduction
In this experiment standing waves will be observed on a vibrating string. The wavelengths of the waves and the tension in the string will be measured. From these measurements and the known frequency of the wave, the mass per unit length of the string will be determined.

The velocity of transverse traveling waves on a stretched string is given by

\[ v = \sqrt{\frac{T}{\mu}} \]

where \( T \) is the tension and \( \mu \) is the mass per unit length of the string. The wavelength is then given by

\[ \lambda = \frac{v}{f} = \frac{1}{f} \sqrt{\frac{T}{\mu}} \]

where \( f \) is the frequency.

Equipment and Method
A traveling wave is generated by a motor attached to one end of the string. The tension in the string is determined by a weight hung from the other end of the string which runs over a pulley. The traveling wave is reflected back along the string at the pulley, and is reflected again at the motor. When the second reflected wave is in phase with the original wave, a standing wave pattern will be observed on the string. The distance between the nodes of the standing wave is half a wavelength.

Procedure
Place a 100g mass on the end of the string. Adjust the position of the motor and/or the motor until a standing wave is observed. Measure the wavelength, \( \lambda \), and estimate the uncertainty.

Repeat the above using at least three other masses connected to the end of the string.

Plot \( \lambda^2 \) versus \( T \), the tension of the string. The slope of this line will give the mass density.

Compare this value with the actual mass density. To do this, measure the length of the string and weigh it, in order to get \( \mu \).

Repeat the above with wire.
Electric Field Plotting

Introduction
Electric fields take on definite forms depending on the source from which they are formed. The fields can be plotted by finding the equipotential lines, lines in which the electric field has the same value. In this lab, we will investigate the electric fields coming from a parallel plate, a dipole, and a dipole plus circle.

Equipment
3 Carbonized Acetate Sheets, 1 dry cell (1.5V), 1 digital multimeter, Galvanometer, 2 voltage probes

Procedure
In the parallel plate sheet connect one terminal of the battery to each plate. Your result will be recorded on Xerox copies of the carbonized sheets. Place one of the voltmeter probes at some point near one of the conducting plates. Move the other probe along the carbonized paper until another point is reached for which the voltmeter reading is zero. Mark the location of these two points on the diagram in your notebook.

Keeping the first probe fixed, move the second probe until another point at the same potential is found. In this way locate enough points to allow you to draw the equipotential line connecting these points. Record the potential line relative to the negative terminal of the battery.

Repeat the above procedure with the first probe located at other points and thus sketch a family of equipotential lines for the entire region between the plates. In each case, record the potential of the line relative to the negative terminal of the battery. When the equipotential lines have been drawn, sketch the family of lines that are perpendicular to the equipotential lines. These are the electric field lines that are perpendicular to the equipotential lines. These lines are the electric field lines for the parallel plate charge distribution. They begin on positive charges and end on negative charges.

Repeat for the dipole and dipole plus circle.

Questions
1. What can you say about the electric potential of each of the two points you first recorded?

2. How will you decide when you have taken “enough points”?

3. How would you find the magnitude of the electric field in a given region of your diagram?

4. What effect does the circle have on the field distribution?
Magnetic Force and Induction

Part 1: Magnetic Force
Set the controls of the oscilloscope so that you see a stationary spot at the center of the viewing screen. Be careful to turn down the INTENSITY control so that the spot is not too bright (i.e. no “halo”). Hold a magnet close enough to the top of the oscilloscope so that you see the pattern move. Do this with the magnet pointing vertically. Repeat with the magnet pointing horizontally on the side of the oscilloscope. In light of $F = qv \times B$, can you identify the north pole of the magnet?

Part 2 – Induction

Equipment – 1 Oscilloscope, 1 Galvanometer, 1 magnetic compass, 1 set of induction coils, 1 DC Tap switch, 1 Bar magnet

Method – In part 1, a Galvanometer G is connected to the terminals of an induction coil. A magnetic field is produced or by a coil inside of a larger coil. The currents induced in the larger coil as the field changes are observed in the Galvanometer.

Procedure
Move the North pole of a bar magnet in and out of a coil connected to the Galvanometer. Record your results. Repeat with South pole.

Pass a DC current through the smaller coil of the induction set.

Place the smaller coil inside the larger coil and pass a DC current through the smaller coil. Observe the magnitude and direction of the current induced in the larger coil when the current in the smaller coil is turned on and off. Repeat with the direction of the smaller coil reversed.

Repeat the second step with an iron rod placed inside the small coil.

Replace the DC power supply with an AC signal generator set to a sine wave of 1000 Hz. Use the oscilloscope to observe the voltage across both the large and small coils.

Questions
1. What effect does the magnetic field have on the coil?

2. Which end is the north pole?
DC Circuits

Discussion
Ohmic components obey Ohms’s law of \( V = IR \), where \( V \) is the potential difference, \( I \) is the current, and \( R \) is the resistance (in this lab, a constant). Standard circuit resistors are ohmic. In general, however, \( R \) may depend on factors such as the temperature, the direction of current flow, or the intensity of light falling on the component.

You will use a voltmeter to measure \( V \), the voltage drop across the component, and an ammeter to measure \( I \), the current flow through the component. You keep in mind that ammeters must be connected in series in the circuits, while voltmeters must be connected in parallel across the circuit component whose voltage drop is to be measured.

Part I – Ohmic Components
Make sure the power supply is switched off. Connect the power supply ammeter, and voltmeter to one of the two resistors on your circuit board. Be sure to record which board you have and the position of the resistor on the board. Have the lab instructor “OK” your setup.

Measure \( V \) versus \( I \) in several steps up to 10 volts. Note that the knob on the power supply can be used to vary the voltage \( V \).

Plot \( V \) versus \( I \). Indicate estimated errors in reading the meters.

Part II – Non-ohmic Components
Repeat the procedure for Part I but using a light bulb instead of a resistor.

Part III – Parallel and Series
Repeat the procedure for Part I except with two resistors set up i) in parallel and ii) in series.

Questions
1. Is the resistor ohmic? What is its value? Can you explain any systematic errors?

2. From your graph of the light bulb, does the resistance increase or decrease with increased temperature? What is the approximate bulb resistance for \( V = 6V \).

3. Do the resistors in parallel and series have the total resistance expected from theory? Why or why not?
Reflection, Refraction, Lenses and Optical Instruments

Introduction
This laboratory is to show that the very simple principles of reflection and refraction can lead to sophisticated ideas about optical instruments. We begin with a ray box that has a slotted mask in front of a light bulb to produce a set of thin beams. The rays lie along a plane of surface (a sheet of paper). Your measurements will consist of pencil marks on a piece of paper lying on this surface, indicating the direction of these rays before and after striking a mirror or glass prism. Then you will progress to the study of lenses, which are nothing more than refractors with curved surfaces. Finally you will construct a simple microscope and telescope from two lenses.

Part I – Light Wave Reflection
A. Use the ray box, a mirror, and protractor to verify that $\theta_{\text{inc}} = \theta_{\text{refl}}$. Do this for at least 3 different incident angles.

B. Show that the (virtual) image produced by a mirror is located as far behind the mirror as the object is in front of the mirror. The method is up to you.

Part II – Light Wave Refraction
A. Snell’s Law – Use the ray box, a glass prism, and protractor to verify Snell’s Law of refraction. Take the index of refraction for air to be 1 and glass to be 1.5. Verify the law by finding the ratio of the sines for at least three different values of the incident angle.

B. Critical Angles – Snell’s Law also tells us that if we reverse things, i.e., let light hit a glass-air boundary then

$$\frac{\sin \theta_{\text{inc}}}{\sin \theta_{\text{refr}}} = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1}{n}$$

Now, $\sin \theta_{\text{refr}}$ can never be greater than one, and the maximum angle is 90°. The critical angle is any angle that the incident angle would have to be greater than one. Since this cannot be, light must be trapped inside the glass; it must be totally reflected. Use your prism to find the critical angle. Compare your measured value with what you would expect using the equation above.

C. Dispersion – Use the prism to observe light dispersion. Which colors are bent the most. From measure angles, deduce $n_{\text{blue}}$ and $n_{\text{red}}$ for your prism.

Part III – Lenses
The apparatus consists of an optical bench, which serves as a convenient holder for objects, lenses and a ground glass screen for locating images. The object is an arrow painted on a piece of ground glass illuminated from behind by a light bulb.
Your set of apparatus should include three converging lenses and one diverging lens. You should devise methods for determining the focal lengths of each lens. The method is up to you. Use whatever method you please, but clearly describe your technique, and argue that it is accurate to at least +/- 1 cm. You can ask the TA for any formulas you may need.

**Part IV – A simple microscope**
Use the two shortest focal length converging lenses to make a compound microscope. Figure out a way to measure the magnification accurate to about 30%. Make a sketch showing the object, image, and positions of your lenses.

**Part V – A simple telescope**
Use the longest and shortest focal length converging lenses to make a compound telescope. Figure out a way to measure the magnification accurate to 30%. Remember that for a telescope the angular magnification is the significant measure of performance. Make a sketch showing the positions of your lenses and the intermediate real image formed by the objective lens.

Note: Figures will be provided in class.
The Photoelectric Effect

Introduction
In this experiment you will use the photoelectric effect to measure the Planck constant $h$. This classical experiment led to the first precise determination of $h$, and in 1926 R.A. Millikan received the Nobel Prize for it.

A phototube is illuminated by light of a known wavelength. Electrons are ejected from the photocathode with some kinetic energy $KE$. They are collected as an anode current unless a variable retarding potential $V$ is large enough to stop the electrons. For a given potential $V$ all electrons with $KE < eV$ will be stopped, and at some value $V_0$ even the fastest electrons with a kinetic energy $KE_{\text{max}}$ will be stopped when

$$KE_{\text{max}} = h\nu - W = eV_0$$

with $\nu$ the frequency of the incident light, and $W$ the work function of the cathode material. By measuring $V_0$ for different wavelengths one can determine $h$.

Apparatus
The set-up will be given in lab and consists of a phototube, a rheostat to adjust the retarding voltage, a mercury arc light source with different color filters, a battery supply to the retarding voltage (measured by a voltmeter), and a 1 MΩ resistor plus a digital voltmeter (DVM) to read the anode current as a voltage across the resistor.

Measurement
Wire the circuit as shown in lab. Choose one of the filters, note the wavelength printed on the filter. Switch on the mercury lamp and position it to illuminate the tube. It should be about 25 cm distant from the phototube and aligned to give the maximum current reading (voltage on the DVM). Use a black cloth to protect the phototube from room light. Be careful NOT TO CHANGE the distance or alignment between the phototube and light source afterwards!

Set power supply to 3V, voltmeter to 2.5 V. Vary the retarding potential from 0 to 3 V in steps of 0.1 V and measure the anode current (determined by the DVM reading the 1 MΩ resistor). Take this measurement twice for each of the three wavelengths.

For each wavelength, determine the stopping potential $V_0$ from a plot of the anode current $I$ versus the retarding potential $V$.

Then plot $V_0$ versus the frequency (3 data points). These points should lie on a straight line. Determine $h/e$ from this line and calculate $h$.

Determine $W$ for the photocathode.
Atomic Spectra

Introduction
A plate with many closely-spaced slits is called a diffraction grating. If the slit spacing is \( d \), then the wavelength of light diffracted by the grating can be determined from the equation

\[
\sin \theta = m \lambda
\]

where \( \theta \) is the angle for maximum diffraction, \( \lambda \) is the wavelength, and \( m \) is an integer.

In this lab, we use a diffraction grating (13,400 lines/inch) to measure several intense wavelengths of the discharge spectrum of hydrogen. The whole apparatus can be called a spectroscope. It consists of the light source, the grating, an optical bench and a meter stick used to determine the angle \( \theta \) in the above equation. Mounted on the meterstick are two adjustable light emitting diodes (LEDs) to locate the position of the spectral lines under study.

Procedure
Turn on the LEDs and the hydrogen spectrum tube. In a darkened room, observe the spectral lines by placing your eye as near as possible to the diffraction grating mounted on the optical bench. Move the LEDs so that a particular line appears to pass through the center of each LED (one to the left and one to the right of the middle of the meter stick). You may adjust the position of the grating the “fine-tune” the placement of the spectral lines relative to the LEDs. The distance between the LEDs is \( x \) and the distance from the grating to the discharge tube is \( y \).

The angle \( \theta \) is related to \( x \) and \( y \) by

\[
\tan \theta = \frac{x}{2y}
\]

Use this equation to determine the angle for each spectral line (do as many values for \( m \) as possible) and then use the first equation to find the wavelengths.

Questions and Analysis
1. Determine the wavelengths for each line in the hydrogen spectrum using the procedure outlined above.