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Beam Monte Carlo

for the
\[ \mathcal{K}_2 \mathcal{K} \]

Long–baseline $\nu$ Oscillation Experiment

or

Modeling an \textit{Existing} Neutrino Beam

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NBI,'00 FNAL
Far site event rate prediction in two steps:

Measure the neutrino flux nearby
Multiple measurements at the KEK site.

Modeling the beam:
Our MC

This is used to extrapolate from the nearby measurement

Overview of K2K Experiment

(Charged particle monitors)

Target and focusing horn
The Sanford-Wang formula as we use it:

\[
E \times \left( \frac{d^3 \sigma}{dp^3} \right) \text{mbarns/GeV}^2 = \sigma_{total} \mathcal{W}_1 P_\pi^{\mathcal{W}_2} \cdot (1 - P_\pi/P_p)
\]

\[
\times e^{-\left( \mathcal{W}_3 P_\pi^{\mathcal{W}_4}/P_p^{\mathcal{W}_5} \right)}
\]

\[
\times e^{-\left( \mathcal{W}_6 \theta_\pi (P_\pi - \mathcal{W}_7 P_p (\cos \theta_\pi)^\mathcal{W}_8) \right)}
\]

for protons of momentum \(P_p\) (GeV/c) to produce pions at momentum \(P_\pi\) and angle \(\theta_\pi\).

For CERN's best fit to all data the parameters for \(\pi^+\) are:

\[
\mathcal{W} = (0.881, 1.01, 2.26, 2.45, 2.12, 5.66, 0.14, 27.3)
\]

The overall normalization from these pion measurements is uncertain at the 20% level.
**The modeling of a neutrino beam**

**Positive pion production in p-Be interactions**

- Local coding of a fit to previous data is used for production of secondaries from beam target interactions.
- GEANT-3.15 transports particles through the target station and decay space.
- Magnetic fields in the focusing elements are calculated by local code.
- Neutrino fluxes at each detector location are tabulated as functions of position and energy.
- The major shielding and beam dump are included.
Production multiplicities of $\pi$ in aluminum

Average production per interaction

$P_p$(GeV/c)

$\pi^+$

$\pi^0$

$\pi^-$

(For reference: the spectrum of protons dealt with)
Spectrum of protons interacting in aluminum

Mean 12.74
ALLCHAN 0.9933E+07

11 GeV/c kinematics used
12.85 GeV/c kinematics

399451 protons in shaded part are from second interactions
**Magnetic fields in the horn**

*(Note: Step size is limited to <1mm for hadrons)*

---

**Absolute field in and near two targets**

- Solid histogram is for 250kA, 3cm target.
- Dashed histogram is for 250kA, 2cm target.

(Dotted histogram what we used previously.)

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**Calculating $\mathbf{B}$ in the target.**

From the AIP handbook:

The current distribution in an infinite cylinder:

$$I(r) = \sigma E \frac{\text{ber}(\sqrt{2r}/\delta) + i \cdot \text{bei}(\sqrt{2r}/\delta)}{\text{ber}(\sqrt{2R_0}/\delta) + i \cdot \text{bei}(\sqrt{2R_0}/\delta)}$$

where “ber” and “bei” are Kelvin’s functions (a type of Bessel function), $r$ and $R_0$ are the radius at a point and the rod radius respectively.

By Fourier decomposing our applied current pulse, we can reconstruct the full evolution of current in the target.

*Note* that it doesn’t really matter since removing all field from the target gives only about a 2% change in the expected near/far ratio.
Exactly what really comes out?

- (ASCII) **Neutrino flux tables** are always written for 3 near detector and 1 SK location. These have 100MeV energy bins and varying radius bins from 25cm at the center to 1m at the edge of the experimental hall.
- By setting an appropriate input card, **“vector files”** of ntuples of all particles that exist at the end of the second horn are written. These can then be input to various smaller monitor MCs.
- Another input card would similarly enable the saving of particles after the beam dump (for muon monitor modelling).

**Note** that using these special modes is useless without changing the GEANT “cut values” at the same time.
(Really flux*cross section)

Neutrino Flux/cm² for $10^{20}$ POT

Note: Even for rather different fluxes the far/near ratio is quite similar
MC error studies

Strategy: Repeat independent MC sets and analyse each
Vary inputs to reflect their systematic errors
Check output event rate prediction ratios

Even the MC statistical error is non-trivial

SK flux prediction for MC samples of various sizes

Arbitrary unit for neutrino flux
Proton aiming studies using muon monitor input

Run monitor MC and compare data
Muon pit center versus proton input offset

Data generally within this range

Input for neutrino MC study
(also correlate width inputs)

A (partial) study of proton targetting and profile width
(analysis total predicted event rates far/near)

Data generally within this range
Studying nuclear model dependence

Refitting old pion data

\[
\chi^2 \equiv \sum_{j \in \text{sets}} \left\{ \sum_{i \in \text{pts}} \left( \frac{\sigma(p_{i,j}, \theta_{i,j}) \cdot N_j - \mathcal{S}(p_{i,j}, \theta_{i,j}, p_B; \mathcal{W})}{\Delta(\sigma(i,j))} \right)^2 + \left( \frac{N_j - 1}{\Delta(N_j)} \right)^2 \right\}
\]

where \( \mathcal{S}(p_{i,j}, \theta_{i,j}, p_B; \mathcal{W}) \) is the Sanford-Wang formula calculated at a given pion momentum and direction \((p_{i,j}, \theta_{i,j})\)

This gives us errors on the parameters.

Choose representative set of ‘models’
Run MC and analyse
Compare to our measurements:

Comparing our MC to our data

<table>
<thead>
<tr>
<th>Entries</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.3</td>
</tr>
<tr>
<td>RMS</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\[
\chi^2 \equiv \left( \mathbf{\bar{t}} - \mathbf{\bar{t}} \right)^T \left( \sigma^2 + \sigma_t^2 \right)^{-1} \left( \mathbf{\bar{t}} - \mathbf{\bar{t}} \right)
\]

Gcalor value: 12.
First analysis using only input from other sources
(Relative) results of 64 alternate nuclear models

Entries: 64
Mean: -0.4
RMS: 1.8

(Integrated over exp. spectrum)

This is refined using information from our measurement
### The bottom line:
*Systematic error on $\nu$ interaction rates*

<table>
<thead>
<tr>
<th>Source</th>
<th>Error 1</th>
<th>Error 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear models</td>
<td>+5%</td>
<td>-5%</td>
</tr>
<tr>
<td>$^9\text{Be} \rightarrow ^{27}\text{Al}$</td>
<td>±1%</td>
<td></td>
</tr>
<tr>
<td>$B$ field</td>
<td>+1%</td>
<td>-2%</td>
</tr>
<tr>
<td>p center &amp; profile</td>
<td>+1%</td>
<td>-2%</td>
</tr>
<tr>
<td>target sagging</td>
<td>+0%</td>
<td>-2%</td>
</tr>
<tr>
<td>Kaon prod. norm.</td>
<td></td>
<td>±2%</td>
</tr>
</tbody>
</table>

\[
N_{\mu}(SK) = N_0 \pm (\text{stats})\left(\frac{V_{SK}}{V_{FD}}\right)^{FD \text{ syst MC syst}} +7\% -8\%
\]

The above is estimated for a 2m integration radius of the flux expected at the position of the fine-grained detector. The same result is obtained at smaller radii, but for R up to 3m, the upper error for MC syst. grows from 7 to 9%. Beam MC statistics for this run contribute less than 1/2% error to the extrapolation.
Summary:

- Neutrino production is modelled with some precision
- Simulations agree reasonably well with data
- Extensive studies of systematic error are underway

We think we understand the beam MC this well:

(For one representative analysis.)

Anti-correlation between bins can help.