Finding the Answer
and
Understanding It

WARNING

- This is a physicist’s treatment of statistics.
- We use statistics like we use math.
  - We make mathematicians cringe.
  - We make statisticians cringe.
Experimental Error

Statements you have heard:

- The temperature is
  \[ 22 \pm 1 \, ^\circ\text{C}. \]

- The charge of an electron is
  \[ 1.602176462 \pm 0.000000063 \times 10^{-19} \, \text{C} \]

- The solar neutrino flux is
  \[ 2.35 \pm 0.02 \, \text{(stat.)} \pm 0.08 \, \text{(sys.)} \times 10^6 / \text{cm}^2 / \text{s} \]
Experimental Error

Data/Expectation

SK–I 1496day 5.0–20MeV 22.5kt  (Preliminary)

\[ \chi^2 \text{ for eccentricity} = 4.7 \quad \text{Confidence Level} = 69\% \]
\[ \chi^2 \text{ for flat} = 10.3 \quad \text{Confidence Level} = 17\% \]
(8−1 d.o.f.) (with sys. err.)
Why include the error estimate?

Compare the information:

SK-I 1496day 5.0–20MeV 22.5kt
(Uncertainty unofficially removed)

χ² for eccentricity = 4.7   Confidence Level = 69%
χ² for flat = 10.3   Confidence Level = 17%
(8−1 d.o.f.) (with sys. err.)
Types of Error

- Random or statistical error
  Error due to random fluctuations in the data. Different trials of an experiment give different measurements.

- Systematic error
  Unknown factors that affect the measurement. There are two important classes.
  1. Unknown factors from other random processes
  2. Unknown factors due to uncertain knowledge
Mistakes are not an Error!

- Do your level best to make sure you do not make a mistake.
- Do not present a result until you are willing to defend it.

However, mistakes do happen.
- If a mistake is proven, acknowledge it publicly.
- If at all possible correct the problem.

Mistakes hurt your scientific credibility
Systematic Error: Type 1

Unknown factors from other random processes

- Usually the result of a calibration measurement
  - The calibration of a thermometer.
  - The calibration of ADC

- The calibration measurement will (should) include an error.
  - This error can be used just like a random error.
Systematic Error: Type 2

Unknown factors due to uncertain knowledge

- Usually the result of incomplete knowledge
  - Approximations
  - Theoretical inputs
- No particularly good way to include...
  - Usually punt and handle like a random error.
  - Make a S.W.A.G. at how well the factor is known.
- Be as “intellectually honest” as possible.
  - Don’t underestimate.
  - It’s just as bad to overestimate.
Systematic Uncertainty is Tricky

“Measured Values” vs Publication Date

There is a natural tendency to get the “right” answer.
What Does the Error Mean?

1.602176462 ± 0.000000063 × 10^{-19} \text{C}

We intuitively know what this means.

- The measured value is 1.602176462
- There is some probability (usually 68%) that the true electron charge is between 1.602176309 and 1.602176525.

  - A statement about our knowledge.
  - Or is it? It depends on the definition of probability.
- The true value cannot be found statistically.
THE TRUE VALUE CANNOT BE FOUND USING STATISTICAL ANALYSIS.
Sample and Parent Distributions

Parent mean = 1.0, Parent sigma = 1.0
Sample approaches Parent

Large Sample Distribution

Entries: 1000000

$\chi^2/n_{df} = 47.4 / 147$

Constant: $0.1600 \pm 0.05 \pm 20.04$

Mean: $0.7805 \pm 0.012 \pm 0.001$

Sigma: $0.9971 \pm 0.7634 \pm 0.03$
Probability: Classical Frequency

Probability is the relative frequency of an event as the number of trials tends to infinity.

- A rigorous, objective definition.
- Matches with our quantum mechanical preconceptions.
- Some strange consequences.
What Does the Error Mean? II

Assume a thermometer with a random uncertainty, $\pm 5$, is used to measure a true $T = 0.1$ K.

- About half measurements are like $T = 1 \pm 5$ K.
- About half measurements are like $T = -1 \pm 5$ K.
- But, about 15% are like $T = -6 \pm 5$ K.

Is this reasonable?

- Yes! Time to define the confidence interval.
Confidence Interval

- Probability is the relative frequency.
- There is a single true value → its probability is undefined.

\[ q_e = 1.602176462 \pm 0.000000063 \times 10^{-19} \text{C} \]

The 68% confidence interval is a member of a set of regions which of which 68% contain the true value.

- In the usual since, there is not a 68% chance that the true electron charge is between 1.602176309 and 1.602176525.
- Consider temperature: \( T = -6 \pm 5 \text{ K} \).
- The true temperature must be outside this interval.
- OK since the average of multiple experiments will get the right answer.
Time to confuse the issue

“Subjective”, “Bayesian”, or “Modern” Definition:

Probability measures the **degree of belief** that an event will occur.

- Matches the colloquial definition.
- Let’s us say: $T = 22 \pm 1 \, ^\circ C \rightarrow$ a 50% chance that $T < 22 \, ^\circ C$
  
  (do not play games... tell me the answer!)

**BUT!**

- Introduces the **subjective** degree of belief.
- Considered **EVIL** by some people.
Distributions

\[ \mu = \int xP(x)dx \]

\[ \sigma = \int (x - \mu)^2 P(x)dx \]
Error Analysis

The art of finding the answer.

- Unbiased estimator for the mean.

\[
\bar{x} = \frac{\sum x_i / \sigma_i}{\sum 1 / \sigma_i}
\]

- Unbiased estimator for the sigma.

\[
\bar{x}^2 = \frac{\sum x_i^2 / \sigma_i}{\sum 1 / \sigma_i}
\]

then,

\[
s = \sqrt{\bar{x}^2 - \bar{x}^2}
\]

The result is reported as \(\bar{x} \pm s\)
Error Propagation

What happens when we need a function of a measurement.

Measure $l$, $w$ and determine the area $A = lw$.

Consider $x(u, v)$ as a function of two measurements $u$, $v$. The best estimate of the mean is

$$\bar{x} = x(\bar{u}, \bar{v})$$

The best estimate of the sigma is

$$\sigma_x^2 = \sigma_u^2 \left( \frac{\delta x}{\delta u} \right)^2 + \sigma_v^2 \left( \frac{\delta x}{\delta v} \right)^2 + \sigma_{uv}^2 \left( \frac{\delta x}{\delta u} \right) \left( \frac{\delta x}{\delta v} \right)$$
We can estimate the correlation term by

$$\sigma_{uv}^2 = (u - \bar{u})(v - \bar{v})$$

- If \( u \) and \( v \) are uncorrelated, the \( \sigma_{uv}^2 \) is zero.
- For simplicity (and often for good reason), we will assume \( u \) and \( v \) are uncorrelated and set \( \sigma_{uv}^2 \equiv 0 \)
- Scientific judgement is required to make this assumption.
Chi-Squared: $\chi^2$

The all purpose tool

$$\chi^2 = \sum \frac{(y_i - y(x_i))^2}{\sigma^2_i}$$

We’ve made $N$ measurements with

- results $y_1, y_2, \cdots, y_N$
- measured at $x_1, x_2, \cdots, x_N$
- with errors $\sigma_1, \sigma_2, \cdots, \sigma_N$

The function $y(x)$ represents a theory for the relationship between $x_i$ and $y_i$. 
Data Analysis in this class

- You are expected to do a full error analysis.
  - Correct propagation of statistical errors
  - Complete analysis of instrumental systematic errors
  - Identification and estimation of any theoretical errors.

- You will need to do use a computer to handle the data.
  - Pick an analysis program and learn how to use it.
  - This may be the single most important thing you learn this semester.

- The analysis must be described in your report.
Analysis and Plotting Programs

MAPLE complete mathematic and numerical analysis tool. (department license)

IDL Interactive Data Language. Good commercial package used in industry. (student version $79)

\( R \) Integrated data manipulation, calculation and display environment. (free)

ROOT The new data analysis framework for HEP/NUCL. (free)

PAW The old popular framework for HEP/NUCL. Idiosyncratic. (free)

ORIGIN Good commercial package used by some groups at SB. (very expensive)

EXCEL Most spreadsheets can fit data, but are crude as a data analysis tools.
Take the data set (about 10,000 points) and plot a histogram of the sample distribution using at least 50 bins.

Fit the sample distribution using \( f(E) = f_0 + f_1 \times e^{\frac{(E - E_0)^2}{2\sigma^2}} \) and determine the best fit value and estimated error for \( f_0, f_1, E_0, \) and \( \sigma \).

Prepare your report on a computer and email the results to me as a pdf or postscript file.

The purpose of this assignment is to understand how to fit a non-linear curve to data. You will need to find an analysis package, or write a program to perform the fit. Graduate students must use latex to write the report.